Status of Draft ANSI X9.44 (& More)

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Outline

- ANSI X9.44 update
- Security proof background
- Scheme details

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ANSI X9.44

- Key establishment schemes based on the integer factorization problem
  - Key transport and key agreement
  - RSA algorithm; also Rabin-Williams?
- Companion to ANSI X9.31 for signatures, ANSI X9.42/.63 for discrete logarithm / elliptic curve problem
- Along with other X9 documents, basis for NIST key management scheme FIPS

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Design Choices

- Encryption schemes
  - Primitives and “encoding methods”
- Key transport schemes
- Key agreement schemes

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Primary Methods

- PKCS #1 v1.5 encryption
  - 1991, no security proof, widely deployed
- RSA-OAEP
  - 1994, loose security proof, in some standards, not deployed
- RSA-KEM, et al.
  - 2001 (and previous), tight security proofs, brand new

Project Evolution

- RSA-OAEP in drafts through May 2001
- PKCS #1 v1.5 added in June 2001 to reflect practice, esp. SSL/TLS, but not for use with AES
- TLS working group decides in August 2001 to use PKCS #1 v1.5 with AES
  - NIST draft guideline reflects decision (TLS_RSA_WITH_AES_128_CBC_SHA)

Current Content

- **Encryption schemes:**
  - PKCS #1 v1.5
  - RSA-OAEP
- **Key transport schemes:**
  - One-pass with one public key
    - reflects S/MIME “recipientInfo”
- **Key agreement schemes:**
  - Multiple-pass with one public key, key confirmation
    - reflects SSL/TLS handshake

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Related Efforts

	PKCS #1 v1.5	RSA-OAEP	RSA-KEM	Other
PKCS #1 v2.0	X	X		
IEEE Std 1363-2000		X		
NESSIE Phase 2		X		
CRYPTREC Eval.		X		
ISO 18033-2 Draft		X	X	RSA-OAEP+

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The Adversary

- An adversary is an algorithm that tries to *break* a cryptographic scheme, i.e., solve a problem that undermines the security of the scheme.
- Examples of such problems:
  - Find the plaintext corresponding to a ciphertext in an encryption scheme
  - Find the underlying secret key used to encrypt messages
  - Find the inverse of an element with respect to a function that is assumed to be one-way
  - Given a one-way function, find two elements with the same image (a *collision*)

Goals of Adversary

For asymmetric schemes, two kinds of security goals are normally considered:

- **Indistinguishability of encryptions (IND)**
  - **Given two messages and the encryption of one of the messages (the *target ciphertext*), it is hard to determine which message is encrypted**
- **Non-malleability (NM)**
  - **Given a target ciphertext *y*, it is hard to find another ciphertext *y'* such that the corresponding plaintexts are "meaningfully related"**

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The Strength of the Adversary

Depending on whether the adversary has access to an oracle performing private key operations, we obtain three basic levels of adversary strength:

- **Chosen Plaintext Attack (CPA, offline attack)**
  - **The adversary can only encrypt messages**
- **Non-adaptive Chosen Ciphertext Attack (CCA1)**
  - **The adversary has access to a decryption oracle until, but not after, it is given the target ciphertext**
- **Adaptive Chosen Ciphertext Attack (CCA2)**
  - **The adversary has unlimited access to a decryption oracle, *except that the oracle rejects the target ciphertext***
    - **The CCA2 model is very general – in practice, adversaries are much weaker than a full-strength CCA2 adversary**
    - **Yet, many adversaries are too strong to fit into CCA1**

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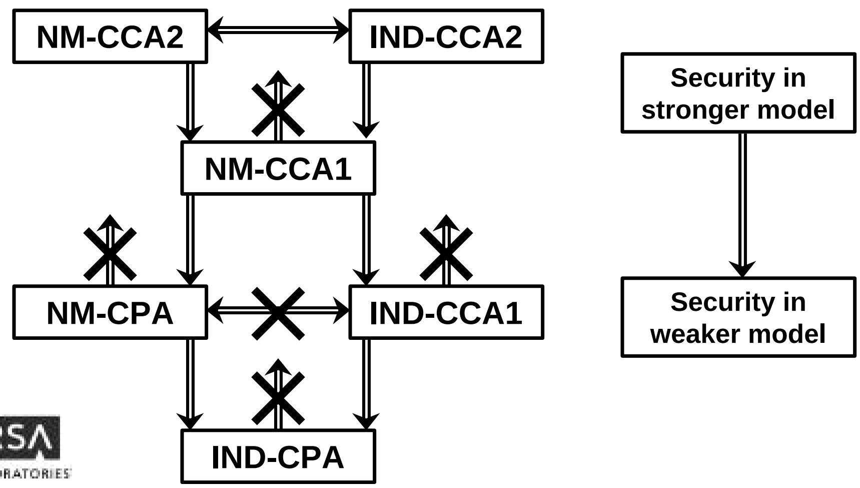
Six Attack Models

The implications between the attack models are as follows:



Security Arguments

Security arguments can be divided into different categories, ranging from the strongest to the weakest:

- Existence of stringent security proof
  - We can *prove* that the scheme is secure under certain assumptions
- Heuristic security arguments
  - We have no proof of security, but we can give evidence that the scheme is hard to break
  - Security claims on symmetric ciphers and cryptographic hash functions belong typically to this category
- Ad hoc arguments
  - "The scheme is secure because there is no known attack"

Assumptions

Given a stringent security proof, there are a variety of possible assumptions on the underlying components, ranging from the strongest to the weakest:

- **No assumptions are needed**
  - **The security proof requires no nontrivial assumptions**
- **A certain mathematical problem is hard**
  - **To break the scheme, you must solve the underlying problem**
- **Some components are assumed to have “ideal” properties**
  - **Examples: Random oracle model; Generic group model**
- **Unconventional restrictions are put on the adversary**
  - **The adversary is prohibited from performing certain operations**

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Hard Problem

- **A typical security proof for an asymmetric encryption scheme ES translates a successful attack into a solution to an underlying hard problem *P* (e.g., the RSA problem).**
- **Typical assumption:**

  P* cannot be solved with probability ε within time *t
- **Desired consequence:**

  ***ES* cannot be broken with probability ε^* within time t^*, where**

  t^* / t is as large as possible;

  $\varepsilon^* / \varepsilon$ is as small as possible.
- **The better t^* and ε^*, the *tighter* security proof**
- **The stronger attack model (e.g., CCA2 instead of CCA1), the smaller t^* and the larger ε^* (if proof even exists)**

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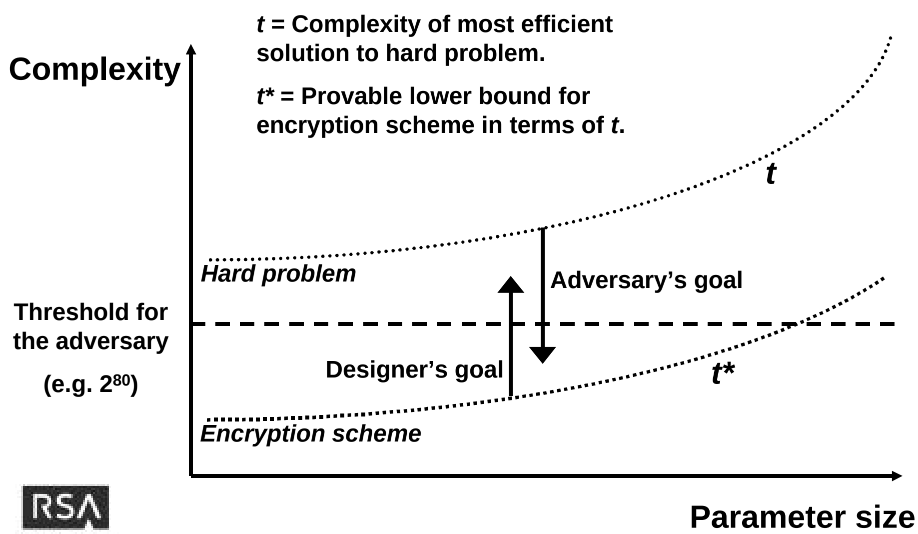
Diagram of Provable Security



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Quality of Security Reduction

- For a security proof to apply to practical parameters, we typically need

  $t^* \approx t$

  $\varepsilon^* \approx \varepsilon$

- Proofs tend to have loose reductions that give useless security guarantees in practice
- Yet, the very *existence* of a security proof with bounds polynomial in *t* indicates that the algorithm design is sound
  - An attack is translated into *one* solution to the underlying problem – not necessarily the most efficient solution
  - The derived solution uses the adversary only as a black box, which may leave room for further optimizations

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Summary

- Four parameters need to be taken into account when analyzing a security proof:
- The challenge for the adversary
  - IND, NM, ...
- Strength of the adversary
  - CPA, CCA1, CPA2, ...
- Assumptions on the underlying primitives
  - Hard mathematical problem, ideal components, ...
- Quality of security reduction (in case there are underlying nontrivial assumptions)
  - Tight, adequate, loose

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Ideal Properties of a Proof

- The challenge for the adversary should be as *easy* as possible
- The adversary should be as *strong* as possible
- The assumptions should be as *weak* as possible
- Quality of security reduction should be as *good* as possible

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Security Proof Template

Assume $\boxed{\text{ASSUMPTIONS on underlying primitive(s)}}$

Then $\boxed{\text{SCHEME}}$ is secure in $\boxed{\text{MODEL}}$

against a $\boxed{\text{CHALLENGE}}$ – $\boxed{\text{ATTACK}}$ adversary

with $\boxed{\text{TIME bound}}$ and $\boxed{\text{SUCCESS bound}}$

given $\boxed{\text{ADDITIONAL constraints}}$

Example

Assume $\boxed{\text{impossible to invert RSA with prob. } \varepsilon \text{ within time } t}$

Then $\boxed{\text{RSA-OAEP}}$ is secure in $\boxed{\text{the random oracle model}}$

against an $\boxed{\text{IND}}$ – $\boxed{\text{CCA2}}$ adversary

with $\boxed{\text{running time at most } t/2 - O(q^2)}$ and $\boxed{\text{desired success probability at least } 4\sqrt{\varepsilon}}$

given $\boxed{\text{at most } q \text{ oracle queries}}$

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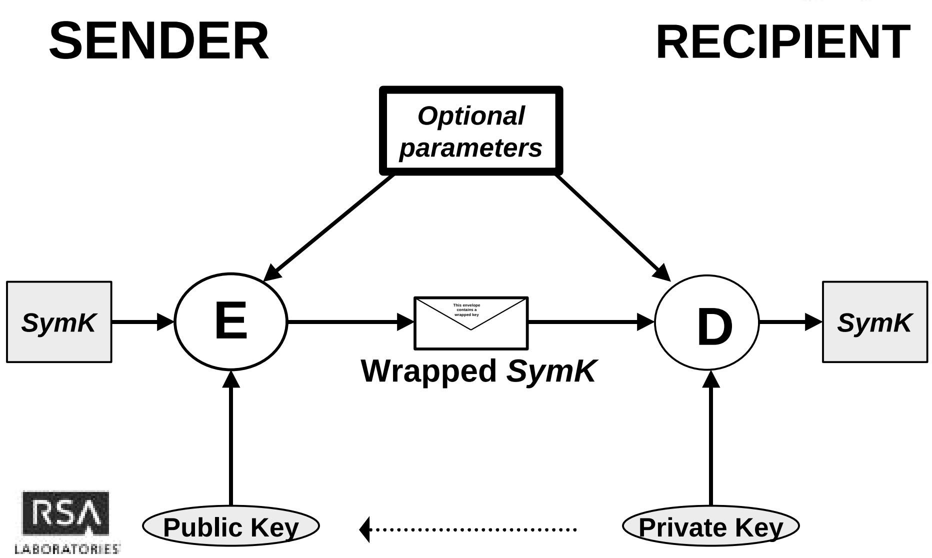
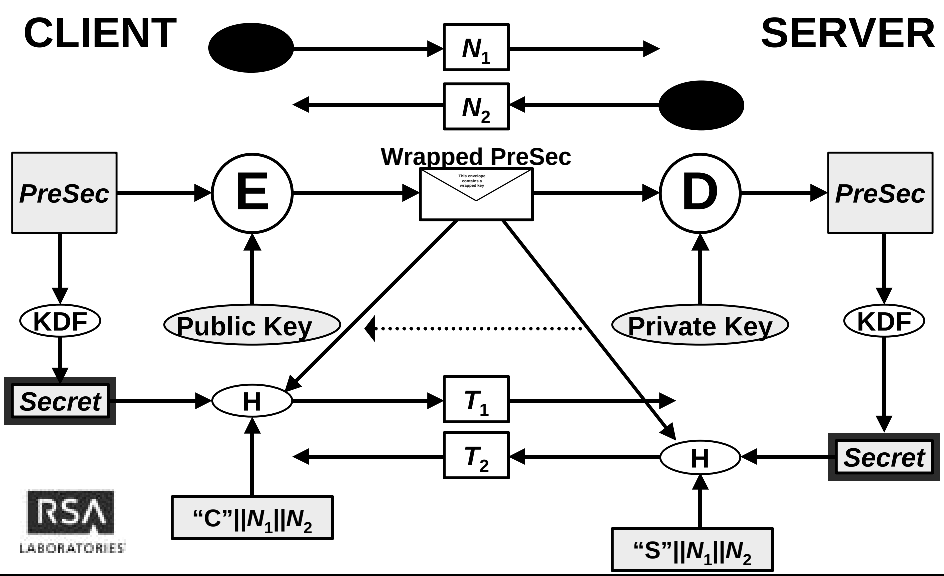
Key Establishment Schemes

- The goal in our setting is one of the following:
  - To transport a key from one entity to another (key transport)
  - To enable two (or more) entities to agree on a key (key agreement)
- Focus on two common schemes:
  - Key transport: Encrypt key with recipient's public key
  - Key agreement: Encrypt key material with recipient's public key; derive key from key material, nonces
- Security depends on underlying encryption scheme

Generic Key Transport Model



Generic Key Agreement Model



Security Requirements

- **For generic key transport, underlying encryption scheme should (ideally) be IND-CCA2**
  - **Wrapped SymK should not reveal information about SymK, given access to decryption oracle at other points**
- **For generic key agreement, underlying "key encapsulation" should be IND-CCA2**
  - **(Wrapped PreSec, T_1) should not reveal information about Secret, given access to decryption oracle at other points**
  - **Freshness, etc. are also important**

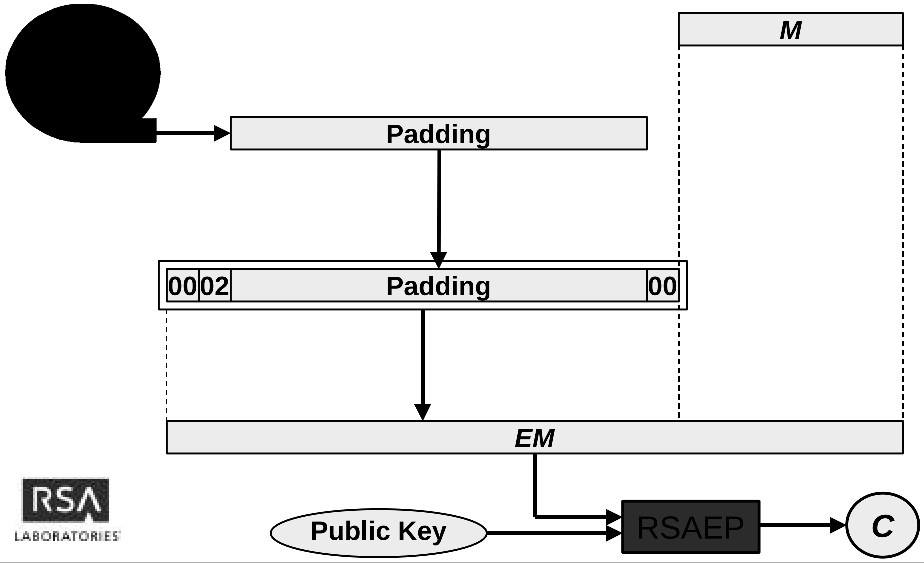
Three RSA-Based Encryption Schemes

- **PKCS #1 v1.5 RSA encryption**
- **RSA-OAEP**
- **RSA-KEM ("simple RSA") + DEM**

RSA-PKCS #1 v1.5

- **Introduced in 1991 in PKCS #1**
- ***De facto* standard for RSA encryption and key transport**
  - **Appears in protocols such as TLS**
- **No security proof exists**
  - **Yet, no fatal attack against the scheme so far**

RSA-PKCS #1 v1.5 Encryption



RSA-PKCS #1 v1.5 Analysis

- Attack against low-exponent RSA when very long messages are encrypted
  - Attack applies if large parts of a plaintext is known or if similar messages are encrypted with the same public key
  - Mounted by Coppersmith et al. (1996); improvements by Coron et al. (2000)
  - Restrictions on the size of the plaintext help thwart attack
  - Not an issue in key agreement protocols
- Chosen ciphertext attack (“Million Message Attack”)
  - Requires a decryption oracle that reports whether a given ciphertext is valid or not
  - For a 1024-bit modulus, the attack requires about one million decryption queries
  - Mounted by Bleichenbacher (1998)
  - Attack is thwarted if ciphertext validity is not revealed, as in TLS

RSA-OAEP

- Asymmetric encryption scheme combining RSA with the OAEP encoding method
- OAEP was invented by Mihir Bellare and Phillip Rogaway in 1994
  - Additional enhancements by Don B. Johnson and Stephen M. Matyas in 1996
- Already widely adopted in standards
  - IEEE Std 1363-2000
  - ANSI X9.44 draft
  - PKCS #1 v2.0 and v2.1 draft

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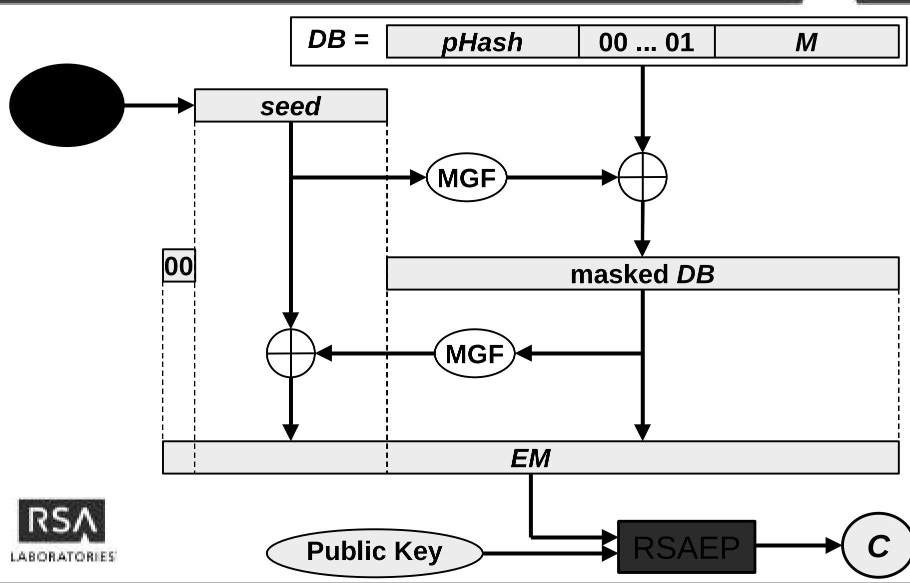
RSA-OAEP Encryption



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RSA-OAEP Security

- RSA-OAEP is provably secure against IND-CCA2 in the random oracle model
  - Fujisaki, Okamoto, Pointcheval, and Stern (2000)
- Assume that the following is true:
  - The RSA encryption primitive cannot be inverted with probability ε within time t
- Then the following holds:
  - RSA-OAEP cannot be broken with prob. ε^* within time t^*, where

    $$\varepsilon^* \approx 4\sqrt{\varepsilon};$$
    $$t^* = t / 2 - O(q^2)$$

    (q is the number of oracle queries)
- Unfortunately, the reduction is not tight

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More on RSA-OAEP

- Bellare and Rogaway proved that RSA-OAEP is IND-CCA1 secure and conjectured IND-CCA2 security
- Shoup observed that a general IND-CCA2 proof for OAEP combined with any trapdoor function cannot be obtained
  - In general, the security of *f*-OAEP can only be related to the hardness of *partially* invert the underlying trapdoor function *f*
- Fujisaki, Okamoto, Pointcheval, and Stern demonstrated that the specific combination RSA-OAEP is IND-CCA2 secure
  - Unfortunately, security bounds are weaker than in the Bellare-Rogaway IND-CCA1 proof

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RSA-KEM+DEM

- KEM = Key Encapsulation Mechanism
- DEM = Data Encapsulation Mechanism
- Construction goes back (at least) to Zheng and Seberry in 1992 and Bellare and Rogaway in 1993. Further development by Victor Shoup
  – RSA-REACT is a variant by Okamoto and Pointcheval
- RSA-KEM (“Simple RSA”) generates a random integer *r*, *derives* a symmetric encryption key from *r* via a key derivation function (KDF), and encrypts *r* with RSA
- DEM encrypts a message *M* with (e.g.) AES using the derived key
  – DEM can be combined with a keyed MAC of *M,* where the key is derived from *r*. The combination is denoted DEM1
  – If *M* is key material, DEM can be AES key wrap

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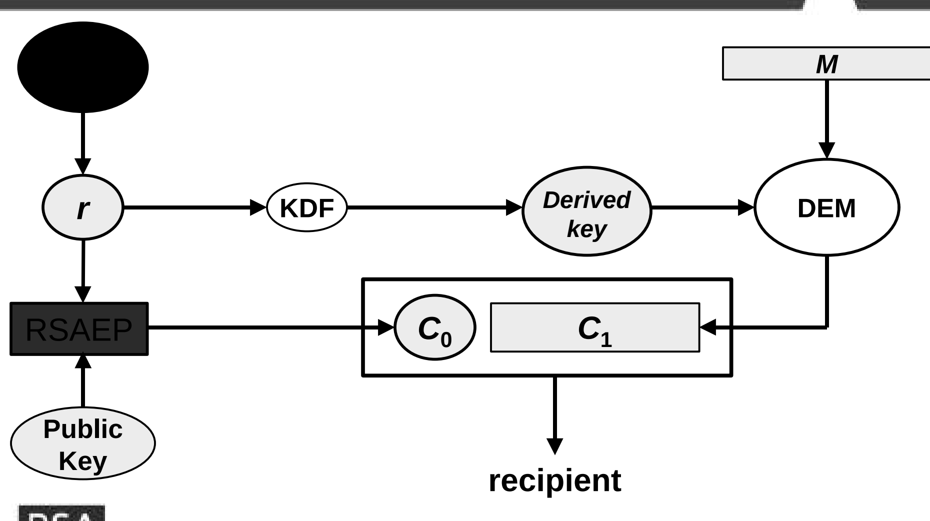
RSA-KEM Encryption



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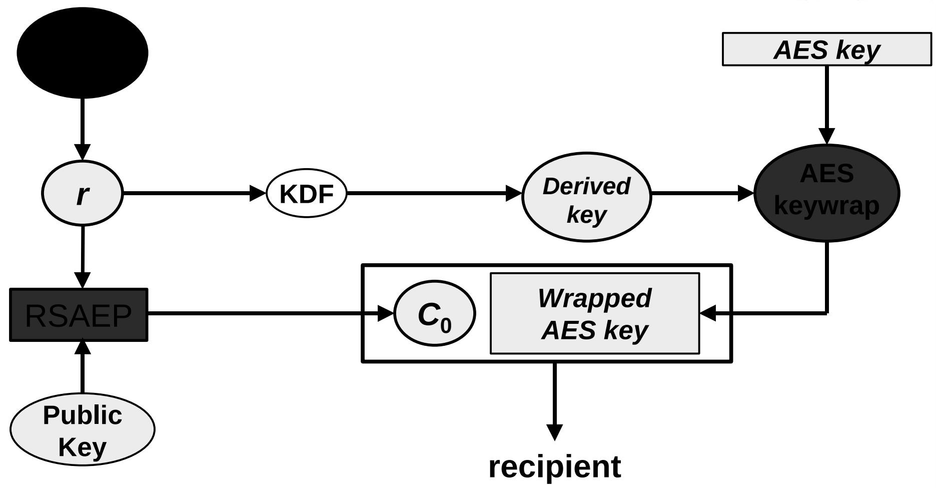
RSA-KEM Key Wrap



Note: KDF can be based on AES via standard block-cipher to hash construction, so only RSA+AES needed

RSA-KEM Security

- RSA-KEM has a tight security, given the random oracle assumption on the KDF; we have

$$\varepsilon^* \approx \varepsilon;$$
$$t^* = t - O(q)$$

  - Reduction is linear in terms of the number of random oracle queries
  - Security proof can be extended to RSA-KEM+DEM1 with the security expressed tightly in terms of the hardness of RSA and the security of the symmetric encryption and MAC algorithms

PKCS #1 v1.5 as a KEM

- **RSA-KEM "encapsulates" keys as**
  - ***K*** **= KDF(*r*),** ***c*** **=** $f_{RSA}(r)$**,** ***r*** **random**
- **PKCS #1 v1.5 (P1) can do so as**
  - ***K*** **= KDF(*r*),** ***y*** **=** $f_{P1}(r)$**,** ***c*** **= (*y*, H(*y*,*r*)),** ***r*** **random**
- **Claim: P1-KEM has tight security under the "Gap-P1" assumption**
  - **Hard to invert** f_{P1} **given a P1 "decision" oracle**
    - **Decision oracle indicates whether (*y*,*r*) is a valid P1 pair, i.e.,** $y = f_{P1}(r)$
  - **TLS handshake using PKCS #1 v1.5 actually based on P1-KEM — so has tight security proof**
    - ***K derived*** **from Secret,** ***y*** **= Wrapped PreSec, H =** T_1**,** ***r = PreSec***

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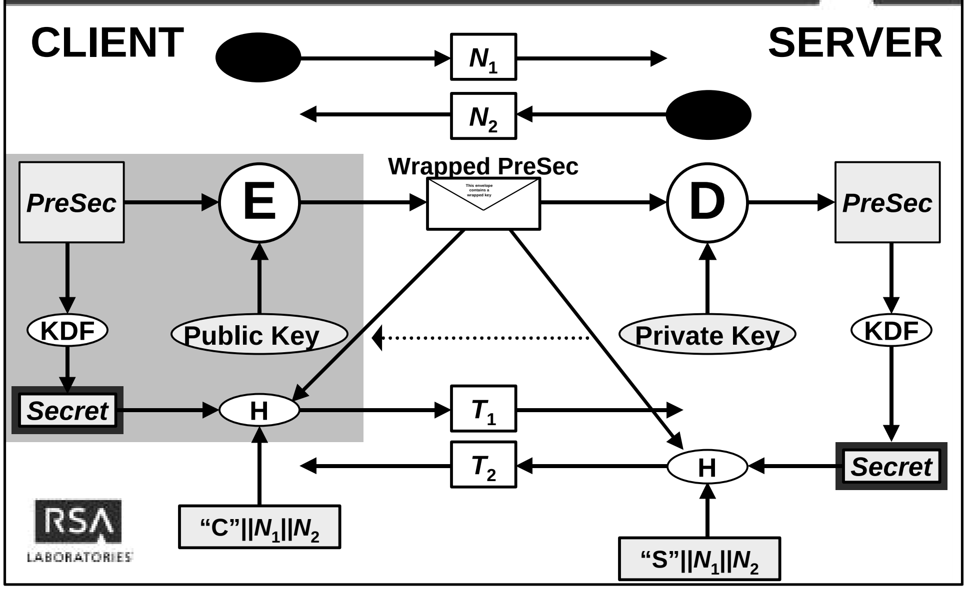
Generic Key Agreement Model



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Conclusion

- ANSI X9.44 draft moving along to guide and reflect practice
- Goal: consider what's in use, what can be proved
- RSA-KEM "key encapsulation" an alternate approach, after PKCS #1 v1.5, RSA-OAEP
- New security claims for PKCS #1 v1.5 key encapsulation, as in TLS

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Backup Slides

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Indistinguishability (IND)

- **Intuition:**
  - Given two messages and the encryption of one of the messages (the target ciphertext), it is hard to determine which message is encrypted
- **The IND adversary works in two steps.**
  - After step 1, the adversary outputs two messages x_0, x_1
  - Let b = 0 or 1 with equal probability. Form a ciphertext y by encrypting x_b and give y to the adversary
  - After step 2, the adversary outputs a bit b' that she believes equals b
  - The adversary is successful if $b = b'$
    - This means that she is able to distinguish between encryptions of x_0 and x_1

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Non-Malleability (NM)

- **Intuition:**
  - Given a target ciphertext y, it is hard to find another ciphertext y' such that the corresponding plaintexts are "meaningfully related"
- **The NM adversary works in two steps.**
  - After step 1, the adversary outputs two messages x_0, x_1
  - Let b = 0 or 1 with equal probability. Form a ciphertext y by encrypting x_b and give y to the adversary
  - After step 2, the adversary outputs a binary relation R and a ciphertext y'
  - Let x' be the decryption of y'. The adversary is successful if $R(x', x_b)$ is true and $R(x', x_{1-b})$ is false

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NM Security ⇒ IND Security

- Let A be an IND adversary. Define an NM adversary B as follows
  - After step 1, A outputs two messages x_0, x_1
    - B outputs the same messages
  - Let b = 0 or 1 with equal probability. Form a ciphertext y by encrypting x_b and give y to B
    - B passes y on to A
  - After step 2, A outputs a bit b'
    - B forms $x' = x_{b'} + 1$, encrypts x' to the ciphertext y', and outputs (y, R), where $R(u,v)$ is true if $u = v+1$
- B is successful if A is successful
  - If $b = b'$, then $x' = x_b + 1$ and x' " $x_{1-b} + 1$

IND and NM Example

In "Pure RSA", a plaintext x is encrypted as $y = x^e \pmod{N}$

Pure RSA does not satisfy the IND or NM criteria:

- NM is violated: Given a ciphertext y, the ciphertext $y' = yk^e \pmod{N}$ has the property that the corresponding plaintexts x and x' satisfy $x' = xk \pmod{N}$
  - This observation exploits the underlying mathematical structure of RSA
- IND is violated: It is easily checked whether or not a certain ciphertext is the encryption of a certain message.
  - This is true for *any* deterministic scheme and also translates into an NM attack
- Conclusion: RSA in itself does not provide any security
  - Yet, it may well be useful as a component in a larger scheme!

NM-CCA2 $\Leftrightarrow$ IND-CCA2

- NM $\Longrightarrow$ IND is always true.
- For the other implication, let B be an NM-CCA2 adversary. Define an IND-CCA2 adversary A as follows.
  - After step 1, B outputs two messages x_0, x_1
    - A outputs the same messages
  - Let b = 0 or 1 with equal probability. Form a ciphertext y by encrypting x_b and give y to A
    - A passes y on to B
  - After step 2, B outputs a ciphertext y' and a relation R
    - A sends y' to the decryption oracle (this is only possible in CCA2, not in CCA1!) and obtains a plaintext x'
    - If $R(x', x_0)$ is true, than A outputs x_0. Else, A outputs x_1
- A is successful if B is successful

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Random oracle model

- A random oracle assumption on a function $H : X \rightarrow Y$ means that an adversary cannot compute or even predict the value of $H(x)$ for any x :
  - To compute $H(x)$, the adversary sends x to a *random oracle*.
  - The oracle responds with a value chosen at random (typically uniformly) from the set Y.
  - The chosen value is independent from earlier queries and responses.
- In practice, a fixed function h cannot be interpreted as a random oracle.
  - Outputs are fixed, not random.
- However, the assumption is useful in that it restricts the model to generic attacks not exploiting the inner structure of H.

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A more realistic oracle model?

- Suppose H is randomized; H takes as input an element x, generates a random r, and outputs $y = H'(r, x)$ with H' fixed.
- Introduce an *inversion oracle* that finds r such that $y = H'(r, x)$ for inputs x and y.
- Drawback: If H' is hard to invert in practice, the inversion oracle cannot be simulated, as opposed to random oracles.
  - The security can only be reduced to the hardness of solving an underlying problem *given* an inversion oracle.
- Possible advantages:
  - H is a fixed (randomized) function even within the model.
  - The problem of inverting H' might be "independent" from the underlying mathematical problem – solving one of the problems may not help in solving the other.
- Model introduced by Gennaro, Halevi, and Rabin.

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Plaintext awareness

- A scheme with IND-CPA security is *plaintext aware* (PA) if an adversary cannot form a valid ciphertext without the corresponding plaintext being derivable from the oracle queries and responses.
  - The adversary has access to an encryption oracle and random oracles but no decryption oracle.
- PA implies IND-CCA2 security.
  - Decryption queries give no information since the adversary already "knows" the plaintext.
- Also, IND-CCA2 does *not* imply PA.
  - In an IND-secure scheme, the public key may leak a valid ciphertext without leaking the corresponding plaintext.
- PA makes sense only in the random oracle model.
  - In the standard model, the adversary can encrypt a plaintext and then "forget" it.

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OAEP Parameters and Options

- **Encoding parameters**
  - **Often empty, but other possibilities exist**
- **Secure hash function**
  - **Applied to the encoding parameters to produce a string *pHash***
  - **Provides plaintext awareness**
- **Mask generation function (MGF)**
  - **Based on a secure hash function (preferably the one applied to the encoding parameters)**
  - **If the MGF is instantiated by a random oracle, the encoded message is (almost) uniformly random and independent from the original plaintext**

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RSA-OAEP+

- **OAEP+ is an adaptation of OAEP introduced by Victor Shoup, replacing *"pHash"* with a hash of a string containing the plaintext and the seed**
- **OAEP+ can be combined with any secure trapdoor function, whereas OAEP is provably secure only with RSA and Rabin**
- **The security reduction for RSA-OAEP+ is better than that for RSA-OAEP; we have**

$$\varepsilon^* \approx \varepsilon;$$
$$t^* = t - O(q^2)$$

- **Yet, this is still not tight; the time bound is quadratic in the number of queries**

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SAEP(+)

- SAEP is a padding method consisting of the first "Feistel round" of OAEP. SAEP+ is derived from OAEP+ in the same manner
  - SAEP introduced by Johnson and Matyas in the early 90s
  - SAEP+ designed by Boneh in 2000
- Rabin-SAEP+ has a *tight* reduction that is *linear* in the number of queries (i.e., $t^* = t - O(q)$)
  - Yet, Rabin schemes are vulnerable to implementation weaknesses that may leak the entire private key
- RSA-SAEP+ has a security reduction roughly equivalent to that of RSA-OAEP

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RSA-KEM+DEM for Key Transport

- RSA-KEM produces a key that can be used to encrypt data
  - Suitable in some situations, but not always
  - Gives a ciphertext overhead of a multiple of 128 bits (in the case of AES) compared to e.g. RSA-OAEP when the message is small
  - Not appropriate in multiparty situations where the same data should be distributed to many entities
    - The same *r* cannot be used more than once
- RSA-KEM+DEM can also be used to encrypt a previously generated key
  - Solves the multiparty problem
  - Fits nicely into existing protocols where the secret key is generated outside the PKE module
  - Yet, still gives ciphertext overhead compared to RSA-OAEP

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